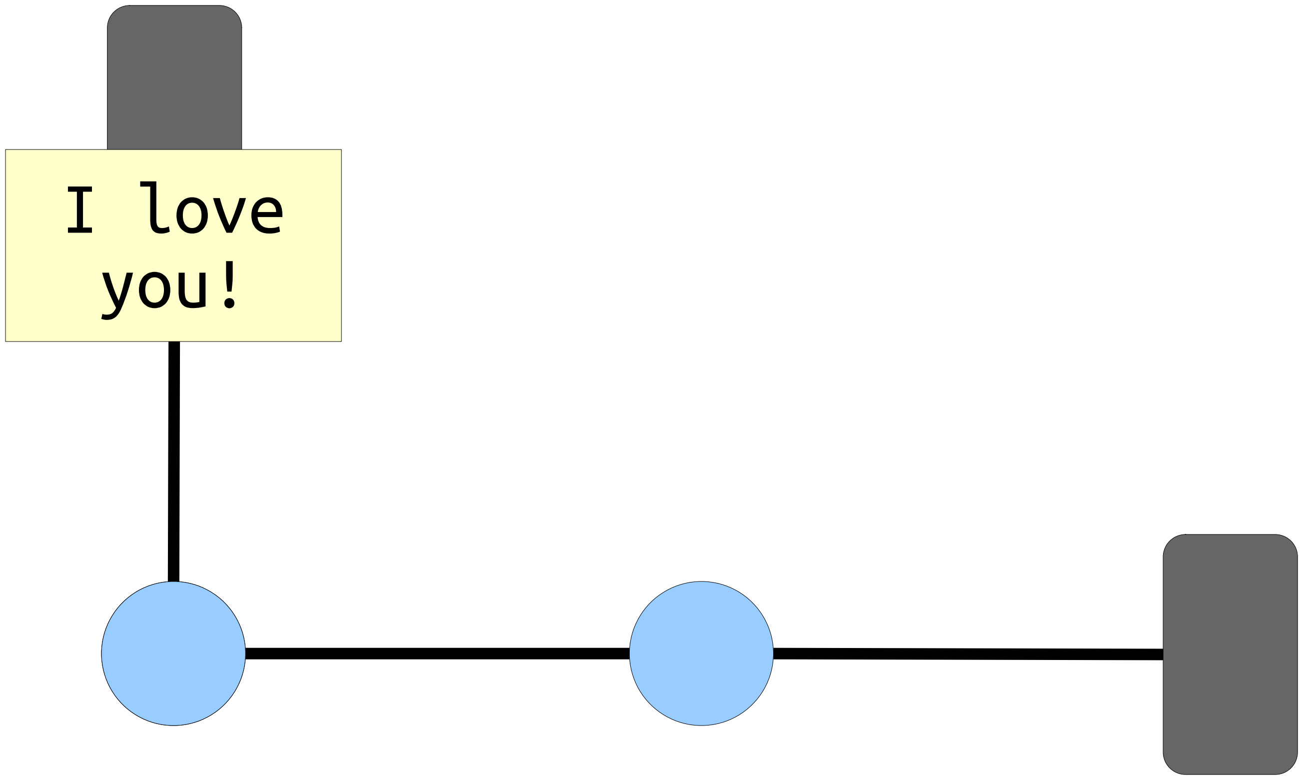
Hashing

Part One

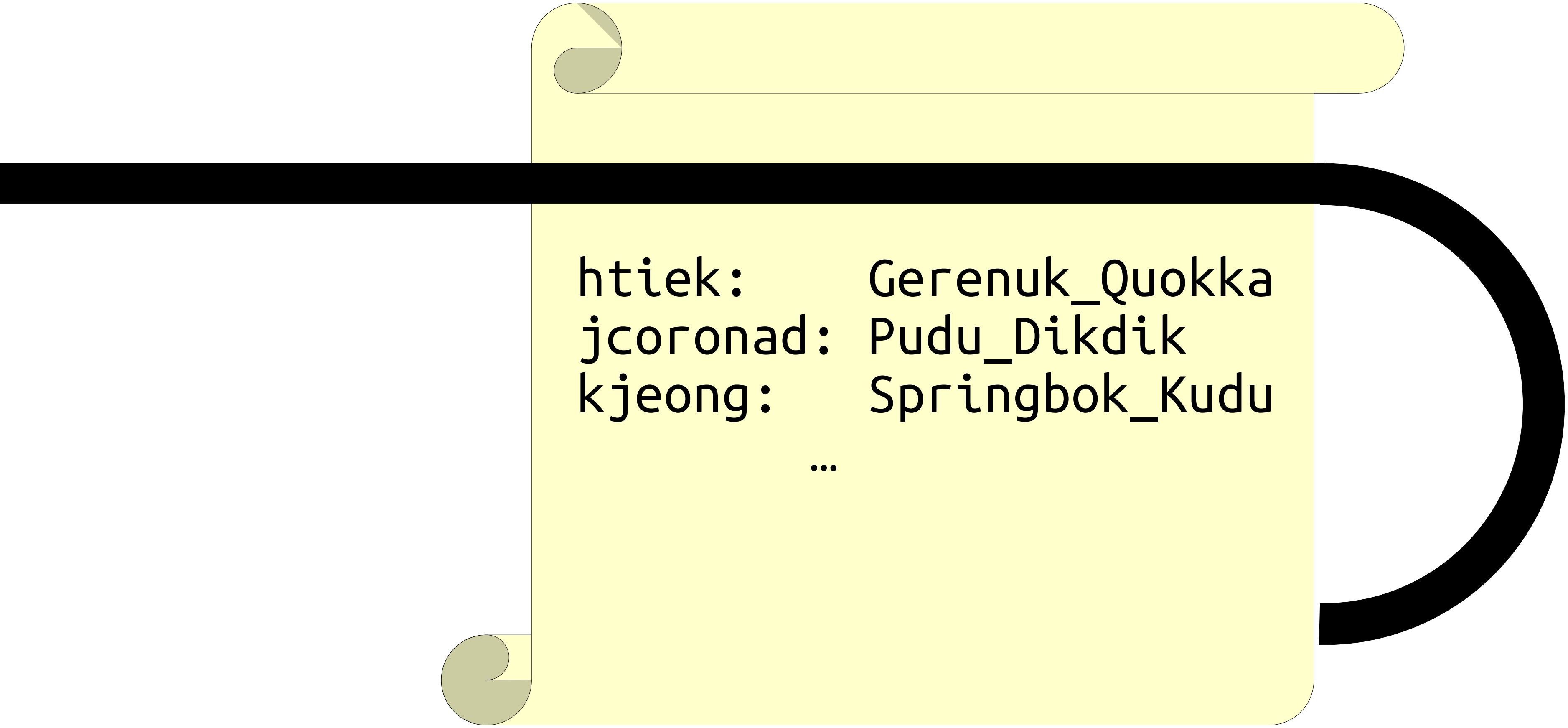
Outline for Today

- ***Hash Functions***
  - An amazingly versatile tool.
- ***Hash Tables***
  - Implementing a very fast `Set`.

Two Motivating Problems



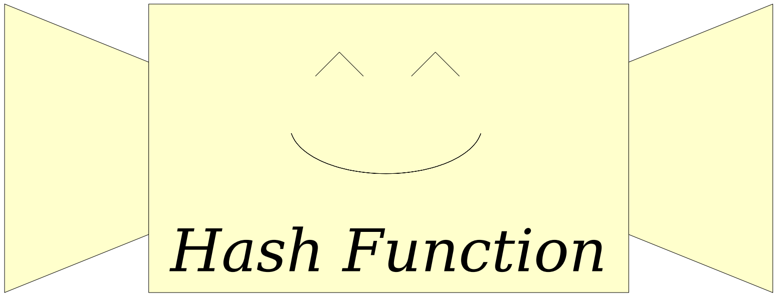
Did my data make it through the network?



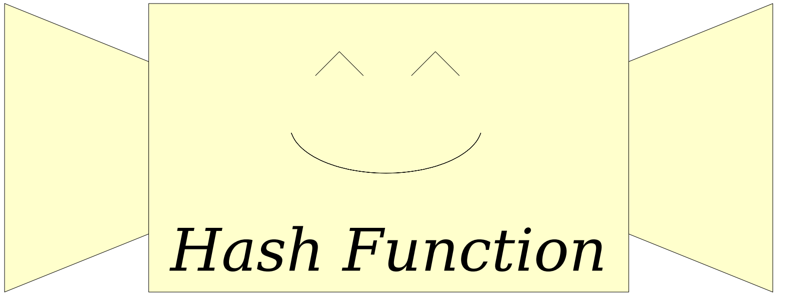
How do servers store passwords?

Way Back When…

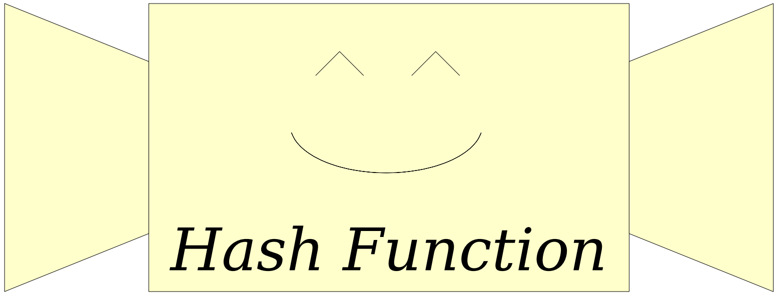
```
int nameHash(string first, string last){
    /* This hashing scheme needs two prime numbers, a large prime and a small
     * prime. These numbers were chosen because their product is less than
     * 2^31 - kLargePrime - 1.
     */
    static const int kLargePrime = 16908799;
    static const int kSmallPrime = 127;

    int hashVal = 0;

    /* Iterate across all the characters in the first name, then the last
     * name, updating the hash at each step.
     */
    for (char ch: first + last) {
        /* Convert the input character to lower case. The numeric values of
         * lower-case letters are always less than 127.
         */
        ch = tolower(ch);
        hashVal = (kSmallPrime * hashVal + ch) % kLargePrime;
    }
    return hashVal;
}
```



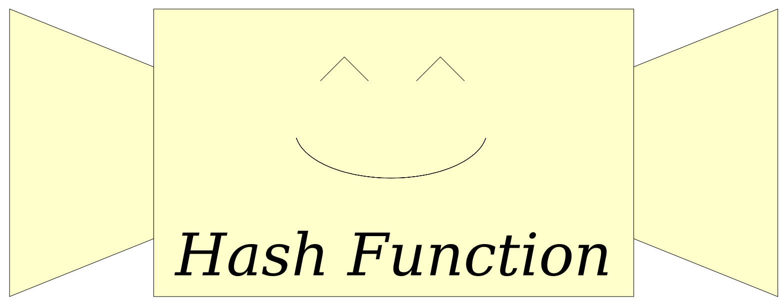
This is a ***hash function***. It's a type of function some smart math and CS people came up with.



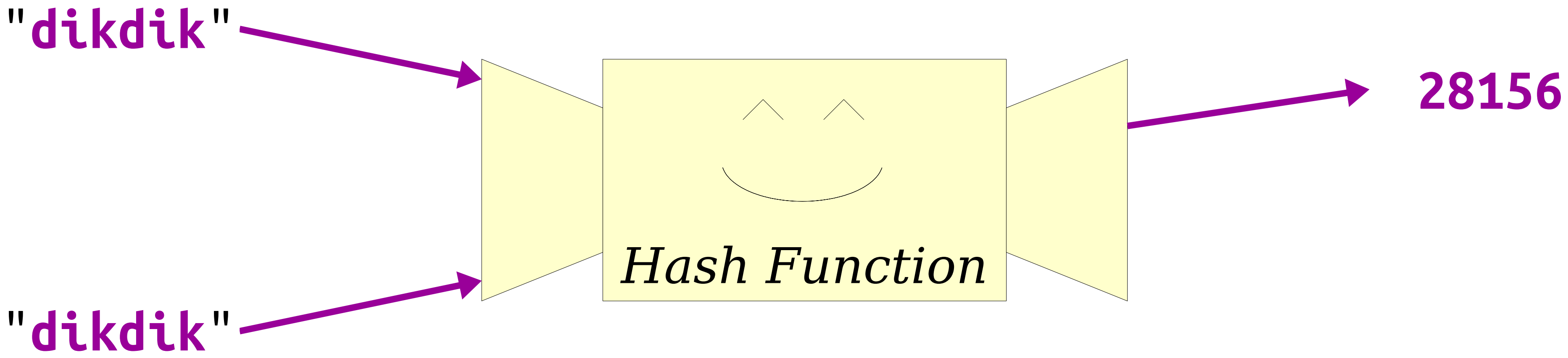
Most hash functions return a number.
In CS106B, we'll use the `int` type.



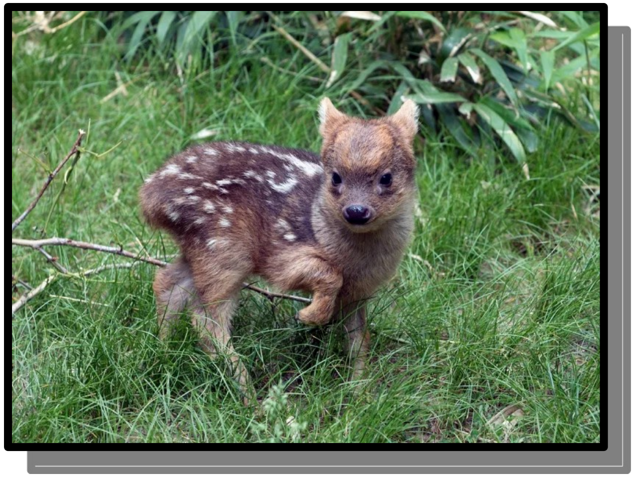
Different hash functions take inputs of different types.
In this example, we'll assume it takes `string` inputs.

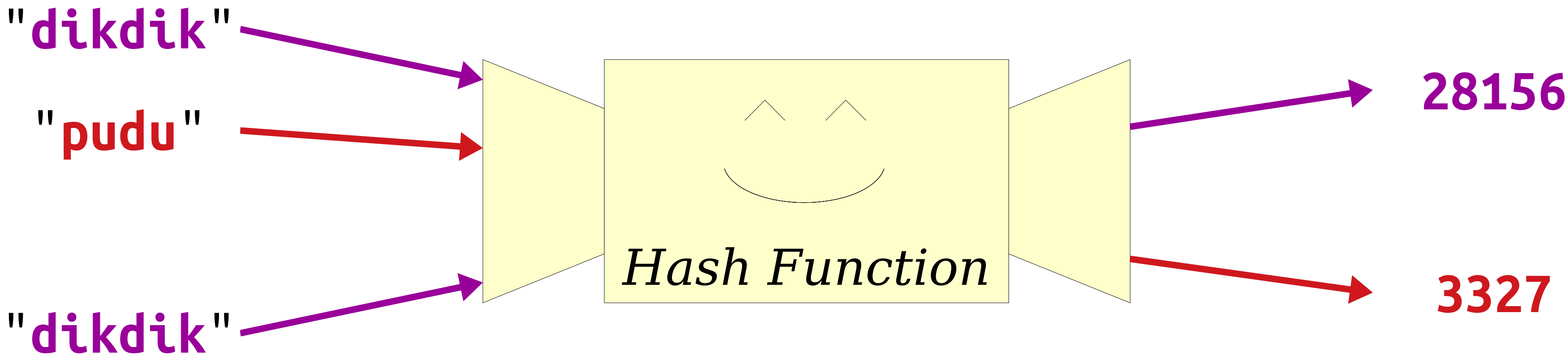


What makes this type of function so special?



First, if you compute the hash code of the same string many times, you always get the same value.





Second, the hash codes of different inputs are (usually) very different from one another.

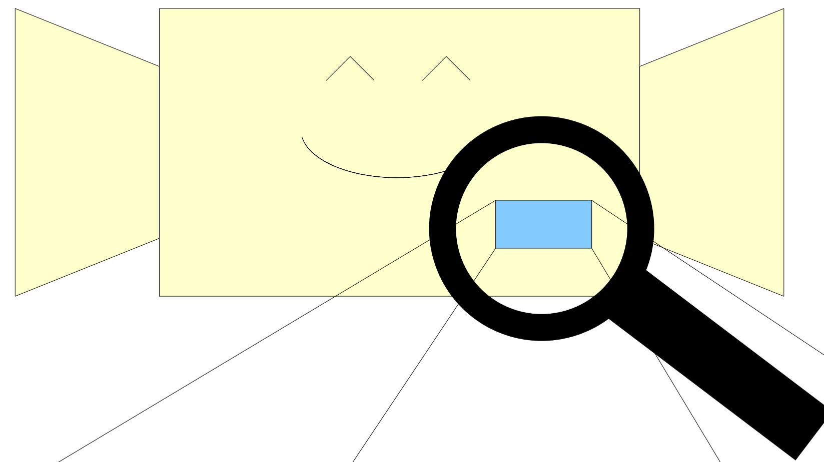
To Recap:

Equal inputs give equal outputs.

Unequal inputs (usually) give very different outputs.

Designing Hash Functions

- Designing good hash functions is challenging, and it's beyond the scope of what we'll explore in CS106B.
- We will assume that some Smart, Attractive, Witty person has created the hash functions we'll use this quarter and won't look into how they work.
- Like finite fields and abstract algebra? Stick around after class and I can share more of the technical details.



$$\Pr_{h \in \mathscr{H}} [h(x) = s \wedge h(y) = t] = \frac{1}{m^2}$$

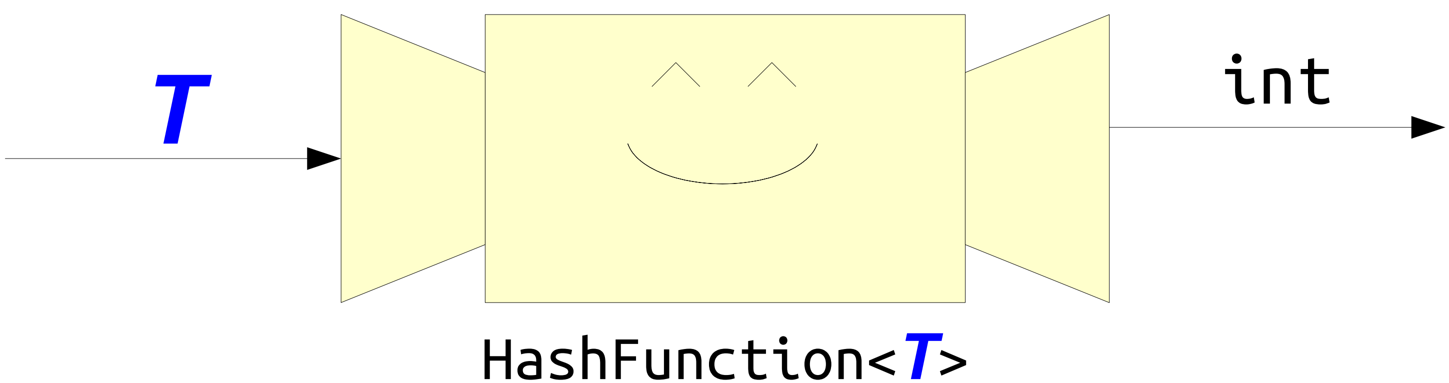
$$h(x_2 x_1 x_0) = T_0[x_0] \oplus T_1[x_1] \oplus T_2[x_2]$$

$$h(x) = \sum_{i=0}^{2} a_i x^i$$

Working with Hash Functions

Working with Hash Functions

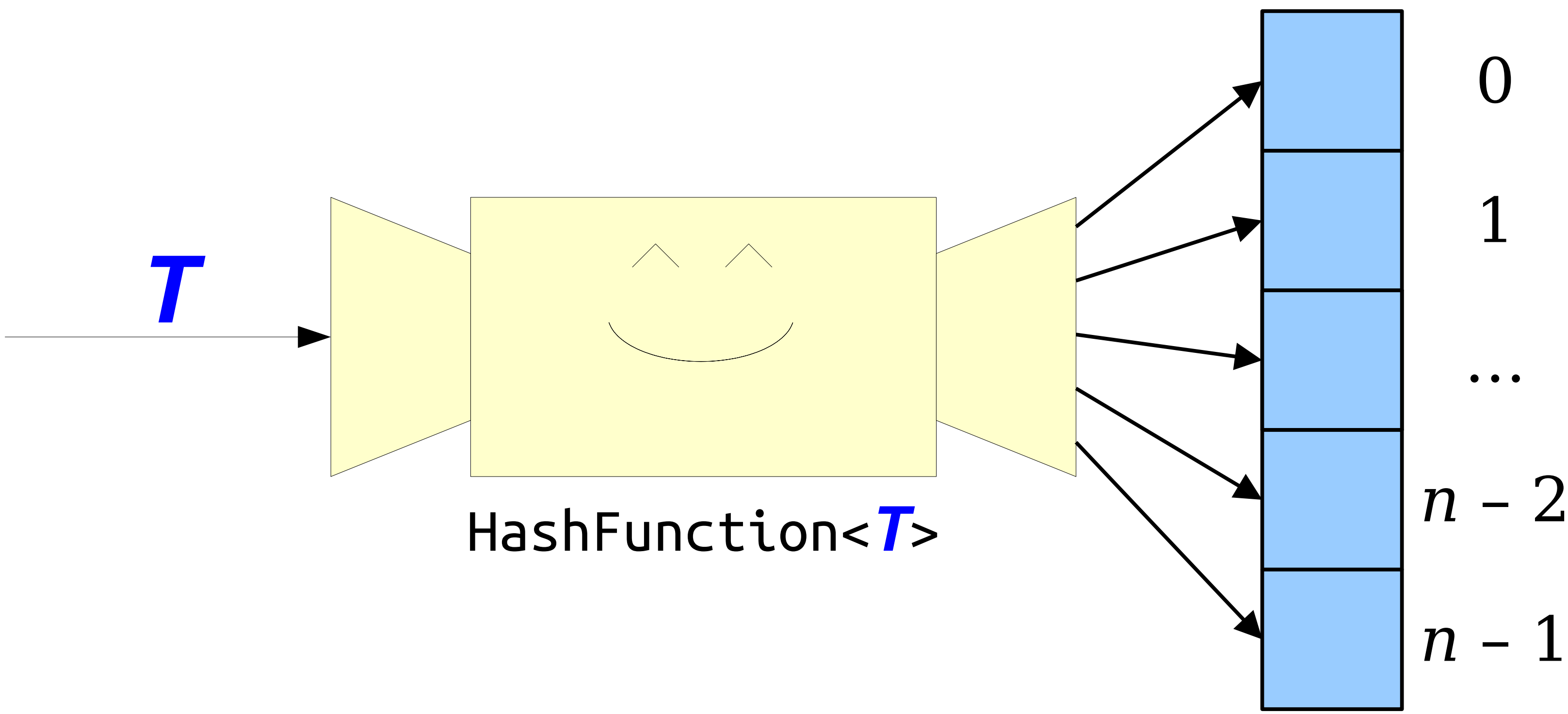
- Every programming language has a different way for programmers to work with hash functions.
- In CS106B, we’ll represent hash functions using the type HashFunction<T>.



Working with Hash Functions

- In many applications, we need a hash function that outputs values in a small range.
- To create a hash function that outputs values between 0 and n – 1, inclusive, use this syntax:

```
HashFunction<T> hashFn = forSize(n);
```



Hash Collisions

- A ***hash collision*** is a pair of inputs to a hash function that produce the same outputs.
- When working with hash functions over a constrained range, hash collisions are unavoidable.
- This isn't the fault of the hash function. If you only have n possible outputs and drop in $n+1$ inputs, you're guaranteed to have a collision.

Hash Collisions

- Think back to the two examples we saw earlier (sending data and storing passwords).
- What bad things might happen in those examples if there are hash collisions?

Answer at
https://cs106b.stanford.edu/pollev

Time-Out for Announcements!

Midterm Debrief

- We graded the midterm exam over the weekend and scores are now available on Gradescope. Solutions and statistics are up on the course website.
- Please reach out to Jonathan, to your section leader, or to me if you want to set up a time to chat about your exam.
- Regrade requests are open. Check EdStem for information about how to submit a request.

Midquarter Check-In

- This part of the quarter can be a stressful time.
- We are all part of a broader campus community and we all need to look out for each other.
- If you’re feeling stressed or overwhelmed, please feel free to reach out to me. I’m happy to help however I can.
- If you know someone in your dorm who’s having a rough time, check in with them and make sure they’re doing okay.
- You are so much more than your academics and your well-being takes precedence over your coursework. If you’re feeling like those are in tension, please reach out to me.

A Note on the Honor Code

Back to CS106B!

An Application:

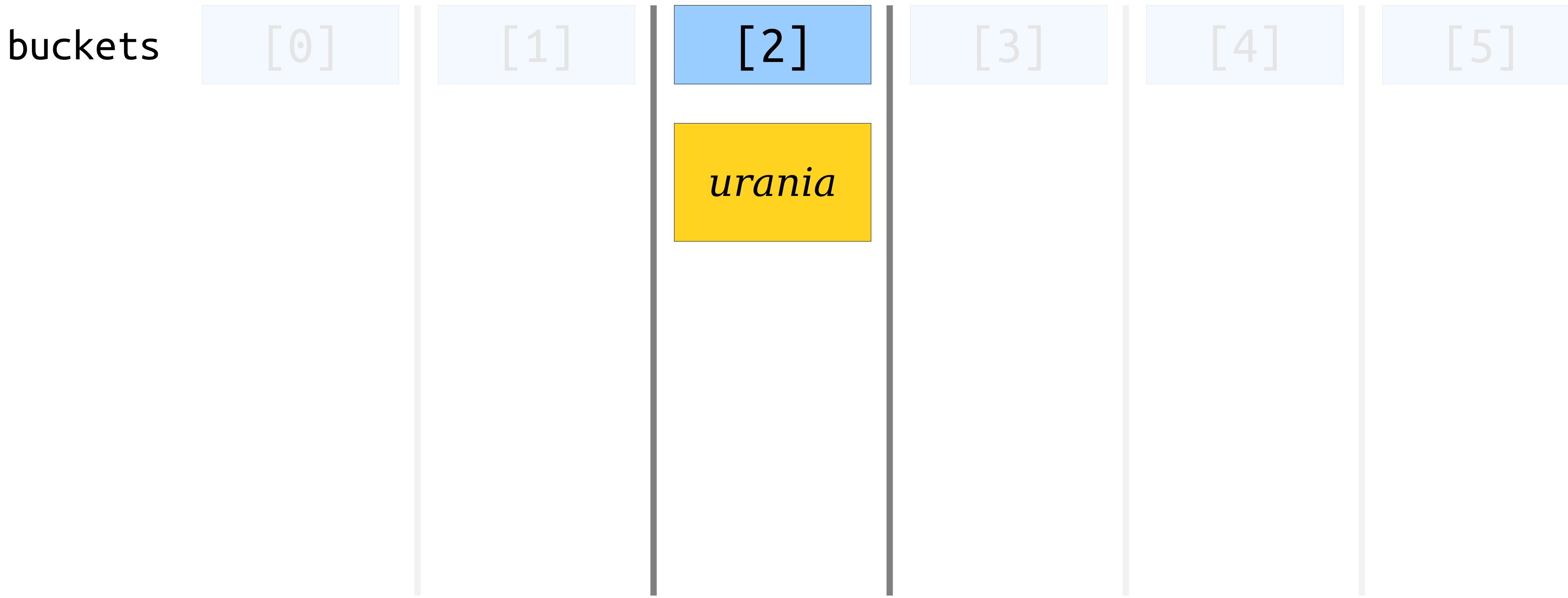
`Map` and `Set`

```cpp
class OurSet {
public:
    OurSet();

    void add(const std::string& str);
    bool contains(const std::string& str) const;

    int  size() const;
    bool isEmpty() const;

private:
    /* What goes here? */

};
```

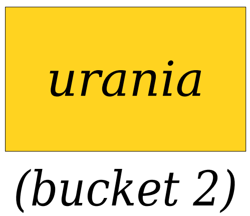
In header files, we refer to the `string` type as `std::string`. It's an Endearing C++ Quirk. Feel free to ask me about this after class if you're curious why.

Our Strategy

- Maintain a large number of small collections called ***buckets*** (think drawers).
- Find a ***rule*** that lets us tell where each object should go (think knowing which drawer is which).
- To find something, only look in the bucket assigned to it (think looking for socks).



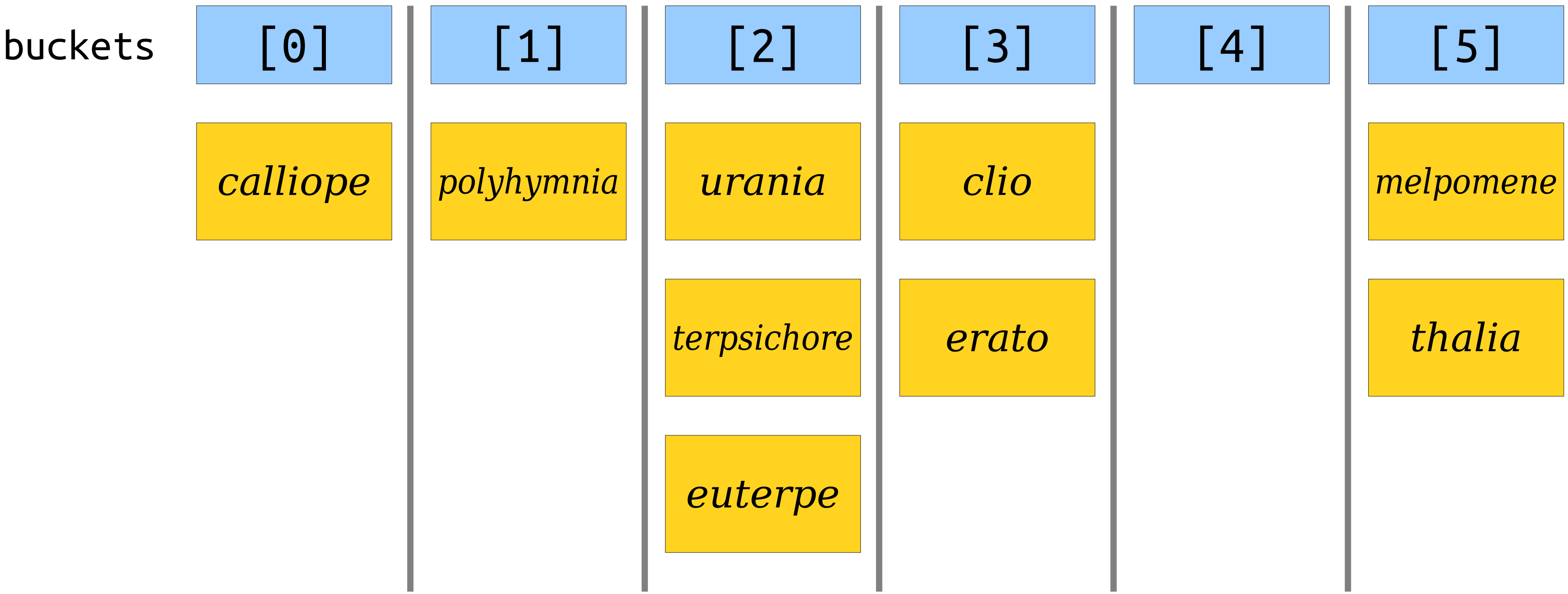

```
void OurSet::add(const string& value) {
    if (contains(value)) return;

    int bucket = hashFn(value);
    buckets[bucket] += value;
    numElems++;
}
```



How efficient is this?

Analyzing our Efficiency

- Each hash table operation
  - chooses a bucket and jumps there, then
  - potentially scans everything in the bucket.
- Choosing the bucket only requires us to hash the input, which is decently quick. So what's needed for that next step?



Analyzing our Efficiency

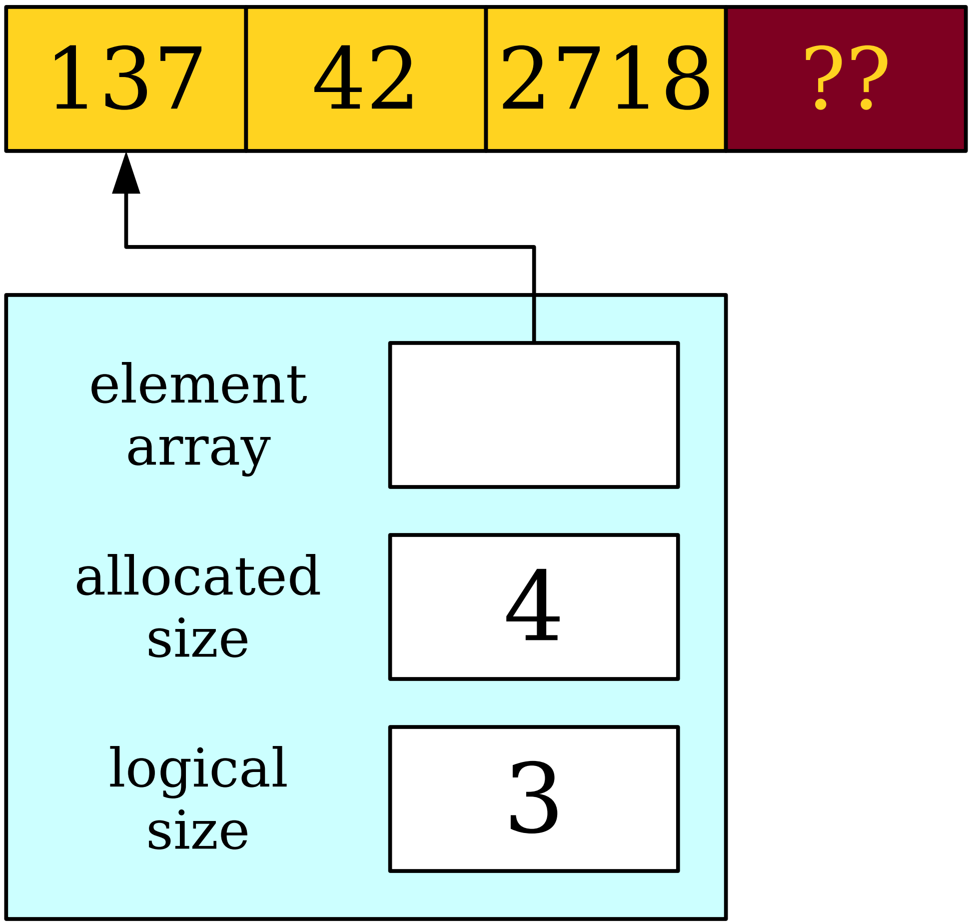
- Imagine we have ***b*** buckets and ***n*** elements in our table.
- On average, we'll have ***n*** / ***b*** items in each bucket.
- The average cost of an insertion, deletion, or lookup is therefore

$$O(1 + n / b).$$

- The ***load factor*** of a hash table, denoted **α**, is the ratio of items to buckets (**α** = ***n*** / ***b***).
- If we keep α small (say, **α** ≤ 2), then the average operation cost is **O(1)**. That's as good as it can possibly get! How do we do this?

Remember When?

- Think back to how we implemented the `Stack`.
- Initially, we had a fixed number of slots.
- Once we ran out of space, we doubled the number of slots and transferred things over.
- ***Idea:*** Whenever α > 2, double the number of buckets. This keeps α below two and makes all operations take average time O(1).



Rehashing

- To perform a ***rehash***, do the following:
  - Get a new list of buckets, twice as big as before.
  - Get a new hash function that distributes elements across the wider range.
  - Redistribute the elements from the old buckets into the new ones, using the new hash function.
  - Use the new buckets and hash functions going forward.
- Time required is O(n). However, this happens so rarely that the extra work averages out to O(1) per insert.

Your Action Items

- ***Work on Assignment 5***
  - If you're following our timetable, by this point you should be done with the Debugger Warmup and String Simulation and making progress through Tone Matrix.
  - Need help or support? Come talk to us at LaIR, in office hours, or over EdStem!

Next Time

- ***Open Addressing***
  - A different conception of hash tables.
- ***Linear Probing***
  - A fast, flexible hash table.
- ***Robin Hood Hashing***
  - A fairer way to hash items.